Wayne
Township Public Schools

Mathematics
Grades 11 - 12
PreCalculus

Dr. Mark Toback, Superintendent
Committee: Andrew Poalillo
Compliance Update Completed June 2022

This curriculum may be modified through varying techniques, strategies, and materials as per an individual student's Individualized Educational Plan (IEP)

Approved by the Wayne Township Board of Education at the regular meeting held on November 15, 2018.

New Jersey Student Learning Standards For Mathematics

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning , strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.

Mathematically proficient students:

- explain to themselves the meaning of a problem and looking for entry points to its solution.
- analyze givens, constraints, relationships, and goals.
- make conjectures about the form and meaning of the solution attempt.
- consider analogous problems, and try special cases and simpler forms of the original problem.
- monitor and evaluate their progress and change course if necessary.
- transform algebraic expressions or change the viewing window on their graphing calculator to get information.
- explain correspondences between equations, verbal descriptions, tables, and graphs.
- draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- use concrete objects or pictures to help conceptualize and solve a problem.
- check their answers to problems using a different method.
- ask themselves, "Does this make sense?"
- understand the approaches of others to solving complex problems.

2 Reason abstractly and quantitatively.

Mathematically proficient students:

- make sense of quantities and their relationships in problem situations.

✓ *decontextualize* (abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and

✓ *contextualize* (pause as needed during the manipulation process in order to probe into the referents for the symbols involved).

- use quantitative reasoning that entails creating a coherent representation of quantities, not just how to compute them
- know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- analyze situations by breaking them into cases
- recognize and use counterexamples.
- justify their conclusions, communicate them to others, and respond to the arguments of others.
- reason inductively about data, making plausible arguments that take into account the context
- compare the effectiveness of plausible arguments
- distinguish correct logic or reasoning from that which is flawed

✓ elementary students construct arguments using objects, drawings, diagrams, and actions..

✓ later students learn to determine domains to which an argument applies.

- listen or read the arguments of others, decide whether they make sense, and ask useful questions

4 Model with mathematics.

Mathematically proficient students:

- apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

✓ In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.

✓ By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

- simplify a complicated situation, realizing that these may need revision later.

- identify important quantities in a practical situation
- map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- analyze those relationships mathematically to draw conclusions.
- interpret their mathematical results in the context of the situation.
- reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students:

- consider available tools when solving a mathematical problem.
- are familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools
- detect possible errors by using estimations and other mathematical knowledge.
- know that technology can enable them to visualize the results of varying assumptions, and explore consequences.
- identify relevant mathematical resources and use them to pose or solve problems.
- use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students:

- try to communicate precisely to others.
- use clear definitions in discussion with others and in their own reasoning.
- state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the context.

✓ In the elementary grades, students give carefully formulated explanations to each other.

✓ In high school, students have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students:

- look closely to discern a pattern or structure.

✓ Young students might notice that three and seven more is the same amount as seven and three more.

✓ Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for the distributive property.

✓ In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7.

- step back for an overview and can shift perspective.
- see complicated things, such as some algebraic expressions, as single objects or composed of several objects.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- notice if calculations are repeated
- look both for general methods and for shortcuts.
- maintain oversight of the process, while attending to the details.
- continually evaluate the reasonableness of intermediate results.

Wayne School District
Curriculum Format

Content Area/ Grade Level/ Course:	Mathematics / 11 - 12 / PreCalculus
Unit Plan Title:	Unit 1: Review of Algebra Unit 1A: Functions and Their Graphs
Time Frame	Enriched: 24 days Regular: 28 days

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Algebra

- Algebra: Arithmetic with Polynomial and Rational Expressions, A - APR
- Algebra: Creating Equations, A -CED
- Algebra: Reasoning with Equations and Inequalities, A -REI
- Functions: Interpreting Functions, F -IF
- Functions: Building Functions, F -BF

Unit Overview

In this unit, students will synthesize and generalize what they have learned about a variety of function families. Students will review knowledge of functions in general, including domain and range. In addition, two or more equations with two or more variables form a system of equations. Students will need to identify different types of equations and choose the most appropriate method to solve the system. Any set of values that makes all equations true in the system is the solution. Systems with two equations in two variables can be solved by graphing, substitution method, or linear combination method. Systems of higher dimensions can be solved by using technology and/or matrices. Systems are often used to model real world problems that require more than one variable.

Standard Number(s)   * i.e: Math: F.LE.A.4   i.e.: NJSLSA.R4.

- A -APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- A -APR.B.2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
- A -APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- A -APR.C.4 Prove polynomial identities and use them to describe numerical relationships. For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity (x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2 can be used to generate Pythagorean triples.
- A -APR.D.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
- A -APR.D.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
- A -CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- A -CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

- A -CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
- A -CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R
- A - REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
- F -IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
- F -IF.C.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.
- F -IF.C.7b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- F -BF.A.1 Write a function that describes a relationship between two quantities.★
- F -BF.A.1b Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
- F -BF.A.1c (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
- F -BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- F -BF.B.4 Find inverse functions
- F -BF.B.4a Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2x^3 or f(x) = (x+1)/(x–1) for x ≠ 1.
- F -BF.B.4b (+) Verify by composition that one function is the inverse of another.
- F -BF.B.4c (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
- F -BF.B.4d (+) Produce an invertible function from a non-invertible function by restricting the domain.

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and
- support different interpretations of real-world phenomena.
- 8.1.12.DA.6: Create and refine computational models to better represent the relationships among
- different elements of data collected from a phenomenon or process.
- 8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original
- and existing algorithms.
- 9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12 prof.CR3.a)

- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
- RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
- RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
- RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

- Standards for Mathematical Practice
  - K-12.MP.1 Make sense of problems and persevere in solving them.
  - K-12.MP.2 Reason abstractly and quantitatively.
  - K-12.MP.3 Construct viable arguments and critique the reasoning of others.
  - K-12.MP.4 Model with mathematics
  - K-12.MP.5 Use appropriate tools strategically.
  - K-12.MP.6 Attend to precision
  - K-12.MP.7 Look for and make use of structure.
  - K-12.MP.8 Look for and express regularity in repeated reasoning.

Intended Outcomes - {Essential Questions}

- Why do we have multiple representations of functions?
- When is a relation considered a function?
- Given any function, how does the function relate to the parent function, algebraically and graphically?
- How do you identify an “even” or “odd” function, algebraically and graphically?
- When is a relation a one-to-one function?
- What are the different ways that we can combine functions?
- What can we determine about the domains of the resulting functions?

Enduring Understandings

- Evaluate functions and find their domains visually and algebraically
- Analyze graphs of functions
- Identify and graph shifts, reflections, and non-rigid transformations of functions
- Find arithmetic combinations and compositions of functions
- Find inverses of functions graphically and algebraically

In this unit plan, the following 21st Century themes and skills are addressed.			
Check all that apply. **21st Century Themes**		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills**	
	Global Awareness	E,T,A	**Creativity and Innovation**
	Environmental Literacy	E,T,A	**Critical Thinking and Problem Solving**
X	**Health Literacy**	E,T,A	**Communication**
	Civic Literacy	E,T,A	**Collaboration**
X	**Financial, Economic, Business, and Entrepreneurial Literacy**		

Student Learning Targets/Objectives (Students will know/Students will understand)

Students will be able to:

- solve systems of equations graphically and algebraically.

  Example: Solve the system of linear equations graphically, by substitution, and by linear combination:

$$x + 4y = -8$$
$$3x + 2y = 6$$

- solve a system of a simple linear and quadratic equation

  Example: Solve the system of equations algebraically. Check the answer with a graphing calculator:

$$y = -5x$$
$$y = x - 2x^2$$

- graph the solution to a system of linear inequalities

  Example: Graph the solution set for:

$$2x + 7y < 14$$
$$x - y \geq 1$$

- represent a real-life problem as a system of equations

  Example: A rental car company charges a flat daily fee plus a charge for each mile driven. A car rented for 5 days and driven for 300 miles costs $178, while a car rented for 4 days and driven for 500 miles costs $197. Find the daily fee and the charge for each mile driven.

- solve an equation *f(x) = g(x)* using technology and finding the point(s) of intersection

Assessments (Pre, Formative, Summative, Other) ***Denote required common assessments with an ****

Assessments – pretests, formative, summative – are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:

A. performance based tasks in support of activities for essential understanding of objectives.
   - I. Projects
   - II. Reports
   - III. Investigations
   - IV. Research

B. other evidence of student learning.
   - I. Class Participation
   - II. *Benchmark Tests/Quizzes
   - III. Teacher Observations

C. varied types of assessment measures to be employed, including rubrics.
   - I. Use of communicators
   - II. Group work
   - III. Clickers
   - IV. Exit Cards
   - V. Homework
   - VI. Teacher quizzes/tests
   - VII. *Common Core Midterm and Final Examinations

Teaching and Learning Activities

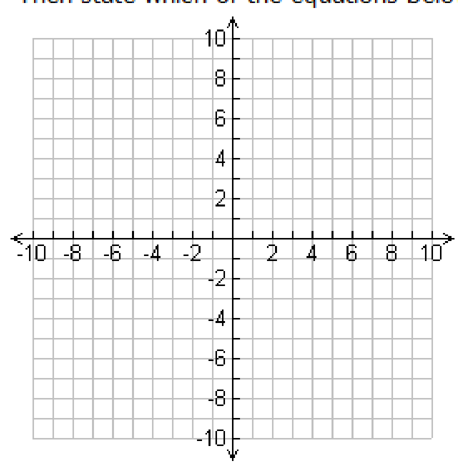
Activities

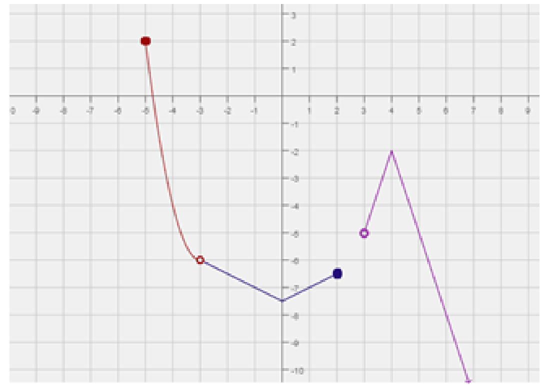
Sample Questions:

1)

On the coordinate grid to the right, graph $y = |x|$. Then state which of the equations below would intersect $y = |x|$ at exactly one point.

- ○ $y = 3$
- ○ $x = 2$
- ○ $y = x - 3$
- ○ $y = x^2 + 3$
- ○ $y = 2x + 4$



	2) Write the piecewise function for the graph at right.  b) State the domain in interval notation: ____________________________ c) State the range in interval notation: ____________________________ d) State over what intervals (if any) the function is: increasing: ________________________________ decreasing: ________________________________ constant: ________________________________ e) Identify any relative maximums or minimums: _______________________________ f) Identify any absolute maximums or minimums: _______________________________ g) Using your graph, tell whether the function is even, odd, or neither: ____________
Differentiation Strategies	● Alternative Assement projects to demonstrate mastery ● Peer to Peer tutoring ● implementations of visual representations ● Technology implementation, use of graphing calcultor ● Allow students to work in small groups ● Provide opportunity for questions Differentiation Strategies for Special Education Students Differentiation Strategies for Gifted and Talented Students Differentiation Strategies for ELL Students Differentiation Strategies for At Risk Students
Honors	N/A (See Unified Mathematics 4 Curriculum)

Resources

- Online applets and interactive virtual manipulatives
  http://nlvm.usu.edu/en/nav/vLibrary.html

- Interactive Math
  http://www.cut-the-knot.org/index.shtml
  http://www.intmath.com/

- Online applets

www.explorelearning.org
www.shodor.org

- Drexel Math Forum
  http://mathforum.org/library/toc.html

- Free Online Calculator
  http://www.coolmath.com/graphit/
  http://www.meta-calculator.com/

- Free online graph paper
  http://incompetech.com/graphpaper/lite/
  http://www.printfreegraphpaper.com/

Content Area/ Grade Level/ Course:	Mathematics / 11 - 12 / PreCalculus
Unit Plan Title:	Unit 1: Review of Algebra Unit 1B: Polynomial and Rational Functions
Time Frame	Enriched: 26 days Regular: 30 days

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Algebra

- Number and Quantity: The Complex Number System, N -CN
- Algebra: Reasoning with Equations and Inequalities, A -REI
- Algebra: Seeing Structure in Expressions, A -SSE
- Functions: Interpreting Functions, F -IF

Unit Overview

This unit develops the structural similarities between the system of polynomials and the system of integers. Students will draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students will connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students will identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Standard Number(s) *** i.e: Math: F.LE.A.4** **i.e.: NJSLSA.R4.**

- N -CN.A.1 Know there is a complex number i such that i^2 = –1, and every complex number has the form a + bi with a and b real.
- N -CN.A.2 Use the relation i^2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- N -CN.A.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- N -CN.B.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
- N -CN.B.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, (-1 + √3i)3 = 8 because (-1+ √3i) has modulus 2 and argument 120°.
- N -CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.
- N -CN.C.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite x2 + 4 as (x + 2i)(x – 2i).
- A -REI.B.4a Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)^2 = q that has the same solutions. Derive the quadratic formula from this form.
- A -REI.B.4b Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.
- A -REI.B.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle x^2 + y^2 = 3.
- A - REI.C.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
- A -SSE.A.1 Interpret expressions that represent a quantity in terms of its context.
- A -SSE.A.1a Interpret parts of an expression, such as terms, factors, and coefficients.
- A -SSE.A.2 Use the structure of an expression to identify ways to rewrite it. For example, see x^4 – y^4 as (x^2)^2 – (y^2)^2 , thus recognizing it as a difference of squares that can be factored as (x^2 – y^2)(x^2 + y^2).
- A -SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
- A -SSE.B.3a Factor a quadratic expression to reveal the zeros of the function it defines.
- A -SSE.B.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- F -IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
- F -IF.C.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.
- F -IF.C.7b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

- F -IF.C.7c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- F -IF.C.7d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and
- support different interpretations of real-world phenomena.
- 8.1.12.DA.6: Create and refine computational models to better represent the relationships among
- different elements of data collected from a phenomenon or process.
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- CRP2. Apply appropriate academic and technical skills.
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- K-12.MP.8 Look for and express regularity in repeated reasoning.

Intended Outcomes - {Essential Questions}

- How can we sketch a polynomial function using its end behavior and zeros?
- Given any function, how does the function relate to the parent function, algebraically and graphically?
- How can we apply the fundamental theorem of Algebra?
- How do you identify an “even” or “odd” function, algebraically and graphically?
- How do we use a graphing utility to analyze polynomial functions?
- How do we know if a rational function has vertical asymptotes?
- How do we find the end behavior of a rational function?

Enduring Understandings

- Sketch and analyze graphs of quadratic and polynomial functions and inequalities
- Max/Min application problems including but not limited to “box problems”
- Use long division and synthetic division to divide polynomials by other polynomials
- Determine the number of rational and real zeros of polynomial functions, and find them
- Perform operations with complex numbers and plot complex numbers in the complex plane
- Determine the domain, range, find asymptotes/holes, and sketch the graphs of rational functions
- Solve rational equations
- Perform operations on rational expressions
- Graph rational functions

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Check all that apply. **21st Century Themes**		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills**	
	Global Awareness	E, T, A	**Creativity and Innovation**
	Environmental Literacy	E, T, A	**Critical Thinking and Problem Solving**
X	**Health Literacy**	E, T, A	**Communication**
	Civic Literacy	E, T, A	**Collaboration**
X	**Financial, Economic, Business, and Entrepreneurial Literacy**		

Student Learning Targets/Objectives (Students will know/Students will understand)

Students will be able to:

- perform arithmetic operations with complex numbers.

Example: Find the values of *a* and *b* given the following relationship:
$(a+3i)+(3b+2i)=7+5i$ and $(9+ai)-(4+bi)=5-i$.

Example: Express $\frac{5+2i}{2-i} - \frac{1-i}{3+4i}$ in $a+bi$ form.

- use complex numbers in polynomial identities and equations.

  Example: Factor $9x^2 + 25$ in the complex field.

  Example: Solve $(x^2 - 1)(x^2 + 4) = 0$ over the complex number system.

- interpret the structure of expressions.

  Example: Prove that the sum of two conjugate complex numbers is a real number.

  $$(a+bi)+(a-bi)=2a$$

  Example: Prove that the product of two conjugate complex numbers is a real number.

  $$(a+bi)(a-bi)=a^2-abi+abi-b^2i^2=a^2+b^2$$

  Example: Factor $x^6 - 64$.

  $$(x^3)^2-(2^3)^2=(x^3-8)(x^3+8)=\cdots$$

- write expressions in equivalent forms to solve problems.

  Example: Determine an equation for the quadratic function with x-intercepts of -1 and 3 with a minimum value of -8.

  $y=2x^2-4x-6$ **or** $y=2(x+1)(x-3)$ **or** $y=2(x-1)^2-8$

- perform arithmetic operations on polynomials.

  Example: What polynomial must be added to $7x^4 - 5x^3 - 8$ to obtain the polynomial $x + 1$?

  Example: From what polynomial must $a^3 - a^2 + 7a$ be subtracted in order to obtain $2a^3 - a + 5$?

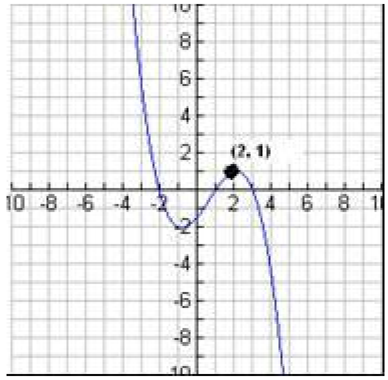
  Example: What polynomial must be multiplied by $x + 3$ in order to produce a product of $2x^3+11x^2+18x+9$?

- understand the relationship between zeros and factors of polynomials.

  Example: If 4 is a zero of $f(x) = 3x^3 + kx - 2$, find the value of k.

  Example: Determine whether or not $x + 1$ is a factor of $x^{100} - 4x^{99} + 3$.

Example: Write an equation for the cubic polynomial graphed below.



- use polynomial identities to solve problems.

  Example: Find the value of $(x - y)^2$ given $x^2 + y^2 = 18$ and $xy = 6$.

- rewrite rational expressions.

  Example: Transform $\frac{-8x^2-14x+11}{2x+5}$ into a sum by dividing.

- understand solving equations as a process of reasoning and explain the reasoning.

  Example: Solve $x + \sqrt{x-2} - 4 = 0$.

  $$\boldsymbol{x - 4 = -\sqrt{x-2}}$$

  $$\boldsymbol{(x-4)^2 = \left(-\sqrt{x-2}\right)^2};$$

  $$\boldsymbol{x^2 - 8x + 16 = x - 2};$$

  $$\boldsymbol{x^2 - 9x + 18 = 0;\ x = 3\ or\ x = 6}$$

  but *x = 6* is extraneous so *x = 3* is the solution.

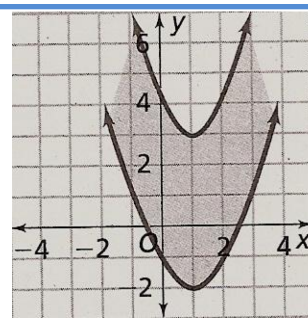
  Example: Solve $\frac{2x-9}{x-7} + \frac{x}{2} = \frac{5}{x-7}$.

  After multiplying each side by $\boldsymbol{2(x-7)\ if\ x \neq 7}$, you have $\boldsymbol{x^2 - 3x - 28 = 0}$.

  So ***x = 7*** or ***x = -4***, but ***x ≠ 7,*** therefore ***x = -4*** is the only solution to the original equation.

- represent and solve equations and inequalities graphically.

  Example: Write a system of inequalities to represent the graph below.



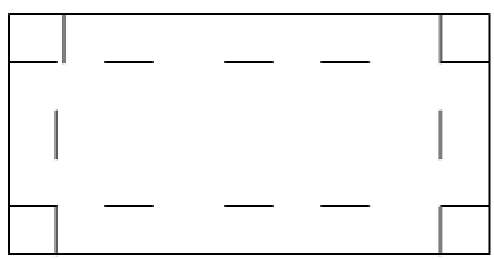
Example: Find all points of intersection between the graphs of $f(x) = x^3 - 6x^2 + 6x + 3$ and $g(x) = -x^2 + 7x - 2.$

- analyze functions using different representations.

Example: Graph $y = x^2 - x,\ y = 6,\ and\ y = x^2 - x - 6$ in the same viewing window. What relationship exists between the points of intersection between $y = x^2 - x\ and\ \ y = 6$ with the zeroes of $y = x^2 - x - 6$?

- interpret functions that arise in applications in terms of a context.

Example: A manufacturer cuts squares from the corners of a 10 cm by 15 cm piece of cardboard and then folds cardboard to make an open-top box.



(a) Label each side length accordingly.
(b) Write the function *V* which gives the volume of the box as a function of *x*? (*V = lwh*)
(c) What are the zeroes of this function?
(d) What is the 'real world' domain of *V*? (What is the length of the biggest length you could make the cut out piece?)
(e) Graph *y = V(x)* showing the turning point and *x*-intercepts of the function on its real-world domain. Sketch what you see by appropriately labelling and scaling your own axes.
(f) Find the appropriate value of *x* that maximizes the volume. What are the dimensions of the open- top box that produces the maximum volume?
(g) What is the maximum volume?

Assessments (Pre, Formative, Summative, Other)	***Denote required common assessments with an ****

Assessments – pretests, formative, summative – are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:

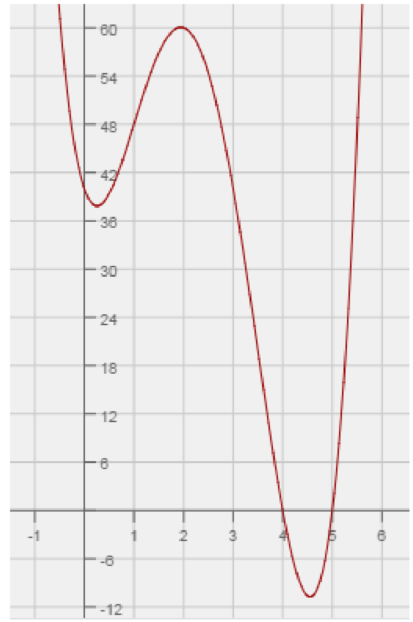
A. performance based tasks in support of activities for essential understanding of objectives.
    I. Projects
    II. Reports
    III. Investigations
    IV. Research

B. other evidence of student learning.
    I. Class Participation
    II. *Benchmark Tests/Quizzes
    III. Teacher Observations

C. varied types of assessment measures to be employed, including rubrics.
    I. Use of communicators
    II. Group work
    III. Clickers
    IV. Exit Cards
    V. Homework
    VI. Teacher quizzes/tests
    VII. *Common Core Midterm and Final Examinations

Teaching and Learning Activities	
Activities	Sample Questions: 1)

Given the function $p(x) = 2x^4 - 18x^3 + 42x^2 - 18x + 40$ and its graph on the right:



a) List all of the possible rational roots of $p(x)$.

b) Using the graph of $p(x)$, which roots on the list from part a) seem to be roots of the polynomial?

c) Using algebra, show your work to confirm that your roots from part b) are true.

d) Write $p(x)$ in factored form:

____

2)

Of the following functions, mark all that have BOTH a horizontal asymptote at $y = 0$ and a vertical asymptote at $x = 0$?

- $t(x) = \frac{2x}{x^2}$
- $k(x) = \frac{2}{x^2-1}$
- $h(x) = \frac{2x^2}{x^2+x}$
- $f(x) = \frac{2x}{x^2-x}$
- $a(x) = \frac{2x}{x^3-8x^2+12x}$
- $u(x) = \frac{2x}{x^2-1}$
- $n(x) = \frac{2x+1}{x}$
- $l(x) = \frac{2x}{x+1}$

3)

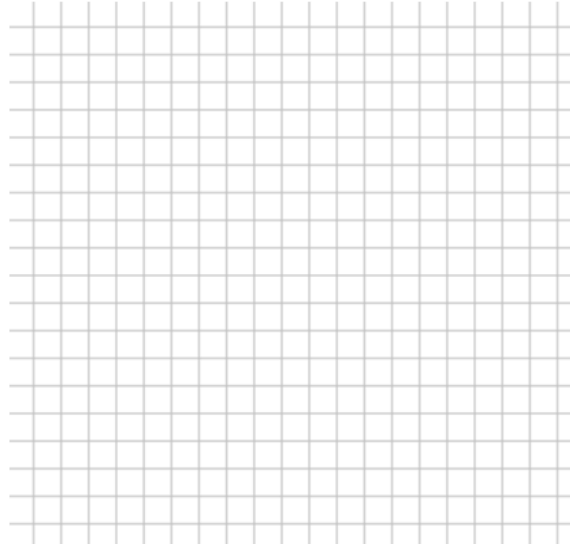
For the rational equations, given a value that *x* approaches, find the corresponding value that *y* is approaching. Then graph each function.

$n(x) = \frac{3x^2+6x}{x^2-4x-5}$

$x \to -1^-,\ y \to$ $\quad$ $x \to -1^+,\ y \to$

$x \to 5^-,\ y \to$ $\quad$ $x \to 5^+,\ y \to$

$x \to -\infty,\ y \to$ $\quad$ $x \to \infty,\ y \to$



Differentiation Strategies

- Alternative Assement projects to demonstrate mastery
- Peer to Peer tutoring
- implementations of visual representations
- Technology implementation, use of graphing calcultor
- Allow students to work in small groups
- Provide opportunity for questions

Differentiation Strategies for Special Education Students
Differentiation Strategies for Gifted and Talented Students
Differentiation Strategies for ELL Students
Differentiation Strategies for At Risk Students

Honors

N/A (See Unified Mathematics 4 Curriculum)

Resources

- Online applets and interactive virtual manipulatives
  http://nlvm.usu.edu/en/nav/vLibrary.html

- Interactive Math
  http://www.cut-the-knot.org/index.shtml
  http://www.intmath.com/

- Online applets
  www.explorelearning.org

www.shodor.org

- Drexel Math Forum
  http://mathforum.org/library/toc.html

- Free Online Calculator
  http://www.coolmath.com/graphit/
  http://www.meta-calculator.com/

- Free online graph paper
  http://incompetech.com/graphpaper/lite/
  http://www.printfreegraphpaper.com/

Content Area/ Grade Level/ Course:	Mathematics / 11 - 12 / PreCalculus
Unit Plan Title:	Unit 1: Algebra Review Unit 1C: Exponential and Logarithmic Functions
Time Frame	Enriched: 13 days Regular: 20 days

Anchor Standards/Domain* ***i.e: ELA: reading, writing i.e.: Math: Algebra**

- Algebra: Reasoning with Equations and Inequalities, A -REI
- Algebra: Seeing Structure in Expressions, A -SSE
- Functions: Interpreting Functions, F -IF
- Functions: Building Functions, F -BF
- Functions: Linear and Exponential Models, F -LE

Unit Overview

In this unit students will synthesize and generalize what they have learned about exponential functions to include solving exponential equations with logarithms. They will explore the effects of transformations on graphs of diverse function, including functions that arise in application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of underlying function. They will identify appropriate types of functions to model a situation, adjust the parameters to improve the model, and compare the models by analyzing the appropriateness of fit and make judgments about the domain over which the model is a good fit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Standard Number(s) * i.e: Math: F.LE.A.4 i.e.: NJSLSA.R4.

- A - REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
- A -SSE.A.1 Interpret expressions that represent a quantity in terms of its context.
- A -SSE.A.1a Interpret parts of an expression, such as terms, factors, and coefficients.
- A -SSE.A.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)^n as the product of P and a factor not depending on P
- A -SSE.A.2 Use the structure of an expression to identify ways to rewrite it. For example, see x^4 – y^4 as (x^2)^2 – (y^2)^2 , thus recognizing it as a difference of squares that can be factored as (x^2 – y^2)(x^2 + y^2).
- A -SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
- A -SSE.B.3a Factor a quadratic expression to reveal the zeros of the function it defines.
- A -SSE.B.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- A -SSE.B.3c Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as (1.15^(1/12))^(12t) ≈ 1.012^(12t) to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
- A -SSE.B.4 Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.★
- F -IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
- F -IF.C.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- F -IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- F -IF.C.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- F -IF.C.8b Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)^t , y = (0.97)^t , y = (1.01)^(12t), y = (1.2)^(t/10), and classify them as representing exponential growth or decay.
- F -IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
- F -BF.B.5 (+) Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
- F -LE.A.4 Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to ab^(ct) = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and

- support different interpretations of real-world phenomena.
- 8.1.12.DA.6: Create and refine computational models to better represent the relationships among
- different elements of data collected from a phenomenon or process.
- 8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original
- and existing algorithms.
- 9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12 prof.CR3.a)
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
- RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
- RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
- RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

- Standards for Mathematical Practice
    - K-12.MP.1 Make sense of problems and persevere in solving them.
    - K-12.MP.2 Reason abstractly and quantitatively.
    - K-12.MP.3 Construct viable arguments and critique the reasoning of others.
    - K-12.MP.4 Model with mathematics
    - K-12.MP.5 Use appropriate tools strategically.
    - K-12.MP.6 Attend to precision
    - K-12.MP.7 Look for and make use of structure.

- K-12.MP.8 Look for and express regularity in repeated reasoning.

Intended Outcomes - {Essential Questions}

- How do we evaluate and graph exponential and logarithmic functions with base a?
- How do you evaluate and graph exponential and logarithmic functions with base e?
- How do you apply the change of base formula?
- How do you use properties of logarithms to evaluate, rewrite, expand or condense logarithms?
- How do you solve exponential and logarithmic equations?
- How do you use exponential growth and decay functions to model and solve real-life problems?
- How do you fit exponential and logarithmic models to sets of data using a graphing utility?
- How do you use exponential and logarithmic functions to model and solve real-life problems?

Enduring Understandings

- Recognize, evaluate, and graph exponential and logarithmic functions.
- Apply exponent rules
- Rewrite logarithmic functions with different bases
- Use properties of logarithms to evaluate, rewrite, and expand, or condense logarithmic expressions
- Solve exponential and logarithmic equations
- Use exponential growth models, exponential decay models, Gaussian models, logistic models, and logarithmic models to solve real-life problems
- Fit exponential and logarithmic models to sets of data

In this unit plan, the following 21st Century themes and skills are addressed.

Check all that apply. **21st Century Themes**		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills**	
	Global Awareness	E, T, A	**Creativity and Innovation**
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X	**Health Literacy**	E, T, A	**Communication**
	Civic Literacy	E, T, A	**Collaboration**
X	**Financial, Economic, Business, and Entrepreneurial Literacy**		

Student Learning Targets/Objectives (Students will know/Students will understand)

Students will be able to:

a) construct and compare linear, quadratic, and exponential models and solve problems.

Example: Sketch $y = x^2$ and $y = 2^x$. Find all values of x for which $x^2 = 2^x$, $x^2 > 2^x$, and $x^2 < 2^x$.

Example: Lucy, Quinn, and Elizabeth are each given $14 on January 1st. The following describes what each lady does with the initial $14 afterward.

- ***Lucy:*** At the end of each month for one year, Lucy simply adds an additional $2 to the initial $14 she received keeping it safe in her piggy bank.
- **Quinn:** Quinn invests her $14 in the stock market where her amount each month is modeled by the quadratic function $A(x) = 0.2x^2 + 0.2x + 14$
- ***Elizabeth:*** Elizabeth invests her money for one year in a bank at a 5% interest rate

compounded continuously.

a) Construct and sketch mathematical models representing each ladies financial situations for the year.

b) Who made the most money? least?

Example: Newton's Law of Cooling states that the difference in the temperatures of a warm body and its cooler surroundings decreases exponentially. Suppose a bowl of soup is 100°C. In a room which is 20°C, its cooling is described by the equation $y = 80(0.875)^t$ where y is the temperature difference in °C between the soup and the room at time t in minutes.

What will be the temperature of the soup after 5 minutes? **61°C**

According to this equation will the soup ever be 20°C? **No**

Example: The power output p, in watts, of a satellite is given by the equation $P(t) = 50e^{-t/250}$ where t is the time in days. If the equipment aboard a satellite requires 15 watts of power, how long will the satellite be operating? ≅ **301 days**

Assessments (Pre, Formative, Summative, Other) ***Denote required common assessments with an ****

Assessments – pretests, formative, summative – are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:

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   VI. Teacher quizzes/tests
   VII. *Common Core Midterm and Final Examinations

Teaching and Learning Activities

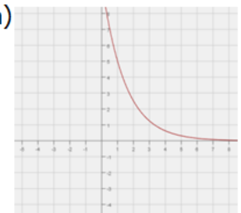
	Sample Questions:

Activities

1)

Multiple Choice Which of the following represent exponential functions?

a)



b) $y = 3 \cdot x^2$

c)

x	-1	0	1	2
y	.75	1.5	3	6

d)

x	-1	0	1	2
y	3	6	9	12

2)

The population of a town increases according to the model $P(t) = 2500e^{0.2197t}$ where t is the time in years, with t = 0 corresponding to 1990. What is the population of the town in 1992, 1995 and 1998? Also, graph the function. On your graph, start at 1990 on the X-Axis and go to 2015.

3)

Solve the following equation:

$$\frac{1}{3}(6)^{-4x} + 1 = 6$$

4)

Determine if each table describes an exponential equation. If it doesn't, name what type of function it does describe. Write your answers on the blanks below each table.

X	Y
-4	-1
-3	-3
-2	-1
-1	5
0	15
1	29
2	47
3	69

X	Y
-4	-1
-3	0
-2	1
-1	0
0	-1
1	0
2	1
3	0

X	Y
-4	$^{1}/_{16}$
-3	$^{1}/_{8}$
-2	$^{1}/_{4}$
-1	$^{1}/_{2}$
0	1
1	2
2	4
3	8

X	Y
-3	54
-2	18
-1	6
0	2
1	$^{2}/_{3}$
2	$^{2}/_{9}$
3	$^{2}/_{27}$

a) ____________ b) ____________ c) ____________ d) ____________

Differentiation Strategies

- Alternative Assement projects to demonstrate mastery
- Peer to Peer tutoring
- implementations of visual representations
- Technology implementation, use of graphing calcultor
- Allow students to work in small groups
- Provide opportunity for questions

Differentiation Strategies for Special Education Students

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Differentiation Strategies for ELL Students

	Differentiation Strategies for At Risk Students
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  www.shodor.org
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  http://www.meta-calculator.com/
- Free online graph paper
  http://incompetech.com/graphpaper/lite/
  http://www.printfreegraphpaper.com/

Content Area/ Grade Level/ Course:	Mathematics / 11 - 12 / PreCalculus
Unit Plan Title:	Unit 2: Trigonometric Functions
Time Frame	Enriched: 35 days Regular: 40 days

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Algebra

- Functions: Trigonometric Functions, F -TF

Unit Overview

In this unit, students are introduced to trigonometric functions. Using right triangles on a unit circle students will derive values of sine and cosine given that the sine of an angle is defined to be the y-value of the coordinate where the angle intersects the unit circle and the cosine of an angle is defined to be the x-value of the coordinate where the angle intersects the unit circle. Students will then learn fundamental identities and be able to evaluate all six trigonometric functions for any angle. Students will be able to the graph all six trigonometric functions, as well as graphing sine and cosine with various transformations. Finally, students will be able to use inverse trigonometric functions in order to find angle values given trigonometric rations.

Standard Number(s)   * i.e: Math: F.LE.A.4   i.e.: NJSLSA.R4.

- F -TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- F -TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- F -TF.A.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for πx, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
- F -TF.A.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

- F -TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
- F -TF.B.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- F -TF.B.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and
- support different interpretations of real-world phenomena.
- 8.1.12.DA.6: Create and refine computational models to better represent the relationships among
- different elements of data collected from a phenomenon or process.
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- 9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12 prof.CR3.a)
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
- RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
- RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
- RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.
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- Standards for Mathematical Practice
  - K-12.MP.1 Make sense of problems and persevere in solving them.

- K-12.MP.2 Reason abstractly and quantitatively.
- K-12.MP.3 Construct viable arguments and critique the reasoning of others.
- K-12.MP.4 Model with mathematics
- K-12.MP.5 Use appropriate tools strategically.
- K-12.MP.6 Attend to precision
- K-12.MP.7 Look for and make use of structure.
- K-12.MP.8 Look for and express regularity in repeated reasoning.

Intended Outcomes - {Essential Questions}

- What are the units that can be used to measure angles?
- What is one radian?
- How are the values of the trigonometric functions on the unit circle derived?
- How can the trigonometric functions of acute angles be evaluated?
- How do reference angles allow us to evaluate angles of any measure?
- How do sketches and transformations of sine and cosine graphs relate to their parent functions and graphs?
- How are the graphs of sine and cosine used to derive the graphs of the other four trigonometric functions?
- What restrictions can be placed on the domains of sine, cosine, and tangent so that they are one to one and their inverses exist?
- How are the inverse trigonometric functions and compositions of trigonometric functions evaluated?
- How can trigonometric functions be applied to real life problems?

Enduring Understandings

- Describe an angle and convert between degree and radian measures
- Identify a unit circle and its relationship to real numbers
- Evaluate trigonometric functions of any angle
- Use fundamental trigonometric identities
- Sketch graphs of trigonometric functions
- Evaluate inverse trigonometric functions
- Evaluate the composition of trigonometric functions
- Use trigonometric functions to model and solve real-life problems

In this unit plan, the following 21st Century themes and skills are addressed.

Check all that apply. **21st Century Themes**		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills**	
	Global Awareness	E, T, A	**Creativity and Innovation**
	Environmental Literacy	E, T, A	**Critical Thinking and Problem Solving**
X	**Health Literacy**	E, T, A	**Communication**
	Civic Literacy	E, T, A	**Collaboration**
X	**Financial, Economic, Business, and Entrepreneurial Literacy**		

Student Learning Targets/Objectives (Students will know/Students will understand)

Students will be able to:

Students will be able to:

- Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Example: The end points of the diameter of a circle are (6, 0) and (−6, 0).

a. What are the coordinates of the center of the circle?

b. A point on this circle has coordinates (2, m). Write possible values for m.

c. Fully explain your answer.

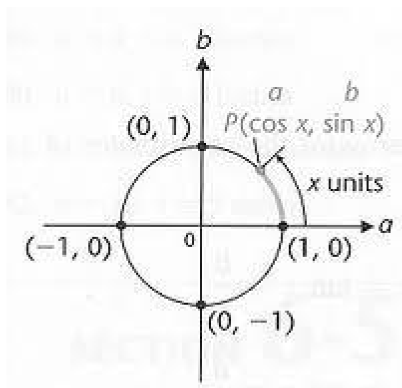
- Extend the domain of trigonometric functions using the unit circle.

Example: A circle has a radius of 1 unit. Give the length of an arc whose measures is:

(a) 360° (b) 180° (c) 90°

Example: Convert 225° to radian measure.

Example: Convert $-\frac{5}{6}\pi$ to degree measure.



Example*: Analyze the diagram above in order to find each of the following:

(a) $\sin\left(\frac{\pi}{6}\right)$ (b) $\cos\left(-\frac{\pi}{3}\right)$ (c) $\sin\left(\frac{7\pi}{4}\right)$

- model periodic phenomena with trigonometric functions.

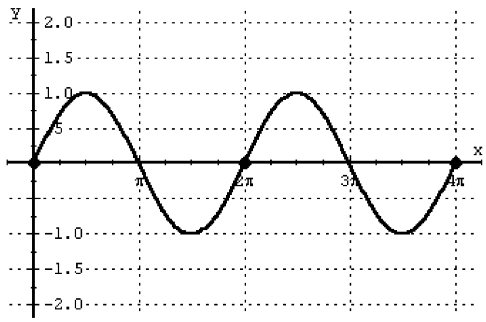
Example: A wheelchair ramp must be built so that it has a slope of $\frac{1}{12}$.

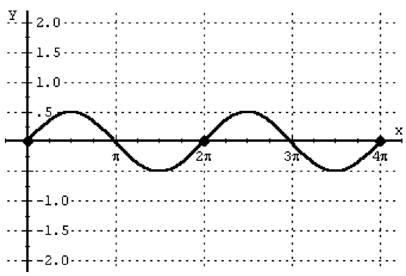
What angle will the ramp make with the horizontal? ≅ **4.8°**.

Example: Suppose a particle moves along the *x*-axis. Its position (*x*-coordinate) at any time *t* seconds where $t \geq 0$ is given by $x(t) = 2\sin(\pi t)$. What is the position of the particle at time $t = 2.3$ seconds? What are the amplitude, period, and frequency of this motion? What is the smallest value of *x* that the particle reaches during its motion?

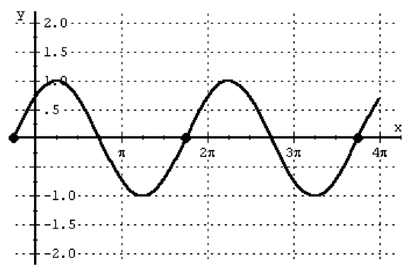
Example: An object hangs from a spring in a stable (equilibrium) position. The spring is pulled 3 feet downward and the object begins to oscillate, making one complete oscillation every 4 seconds. What is an equation of the motion of this object?

Example: From the graph of the parent function *y = sin(x)* shown below, write an equation for a periodic function that models the graphs indicated in (a) – (d).

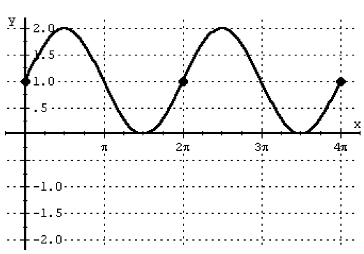




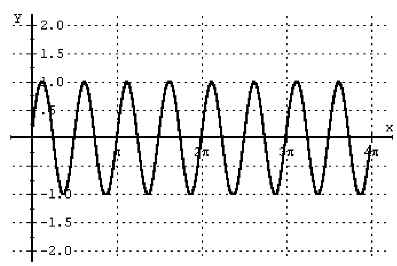
(a)



(b)



(c)



(d)

- evaluate trigonometric expressions.

Example: Sketch the angle θ whose terminal side in standard position passes through the point (-3, 4) and find sin θ, cos θ and tan θ.

Example: Find cos α and tan α given

$sin\,\alpha \;=\; -\frac{5}{13}$ and $\frac{3\pi}{2} < \alpha < 2\pi.$

- evaluate inverse trigonometric expressions.

  Example:

  Find the value of each of the following.

  $\cot\left(\cos^{-1}\left(-\frac{4}{5}\right)\right)$ $\qquad\qquad$ $\tan\left(\sin^{-1}\frac{1}{2}\right)$

Assessments (Pre, Formative, Summative, Other)	***Denote required common assessments with an ****

Assessments – pretests, formative, summative – are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:

A. performance based tasks in support of activities for essential understanding of objectives.
   I. Projects
   II. Reports
   III. Investigations
   IV. Research

B. other evidence of student learning.
   I. Class Participation
   II. *Benchmark Tests/Quizzes
   III. Teacher Observations

C. varied types of assessment measures to be employed, including rubrics.
   I. Use of communicators
   II. Group work
   III. Clickers
   IV. Exit Cards
   V. Homework
   VI. Teacher quizzes/tests
   VII. *Common Core Midterm and Final Examinations

Teaching and Learning Activities

	Sample Questions:

Activities	1) What angle will intersect the unit circle at the point $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$? 2) If $\sec\theta < 0$ and $\csc\theta > 0$, name the quadrant in which the angle θ lies. 3) Which of the following expressions has the same value as $\cot\frac{\pi}{4}$? Mark all that apply! ○ $\sin\frac{\pi}{2}$ ○ $\sec 60°$ ○ $\sec\frac{\pi}{2}$ ○ $\cos 0°$ ○ $\cos\pi$ ○ $\tan\frac{3\pi}{4}$ 4) Fill in the blank with <, > or = to make the statement true. 14) $\sin 164°$ ____ $\sin 165°$ 15) $\sec\frac{\pi}{10}$ ____ $\sec\frac{19\pi}{10}$ 16) $\csc 40°$ ____ $\csc 50°$ 17) $\tan\frac{\pi}{2}$ ____ $\cot\frac{3\pi}{2}$ 5)

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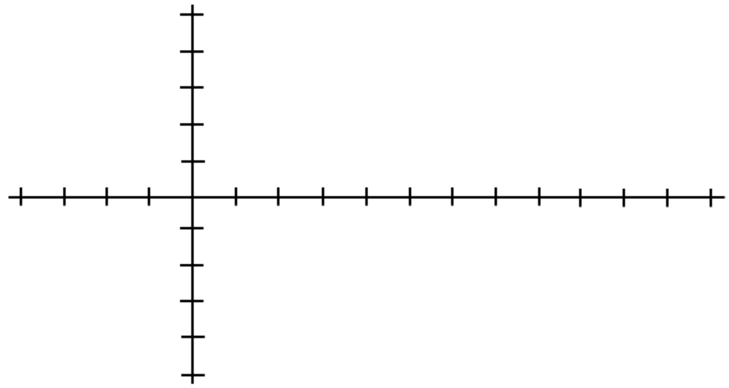
Graph at least two periods of the following.

$$b(x) = 2\sin\left(\frac{1}{2}(x - \pi)\right)$$



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Differentiation Strategies

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- Alternative Assement projects to demonstrate mastery
- Peer to Peer tutoring
- implementations of visual representations
- Technology implementation, use of graphing calcultor
- Allow students to work in small groups
- Provide opportunity for questions

Differentiation Strategies for Special Education Students
Differentiation Strategies for Gifted and Talented Students
Differentiation Strategies for ELL Students
Differentiation Strategies for At Risk Students

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Honors

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N/A (See Unified Mathematics 4 Curriculum)

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Resources

- Online applets and interactive virtual manipulatives
  http://nlvm.usu.edu/en/nav/vLibrary.html

- Interactive Math
  http://www.cut-the-knot.org/index.shtml
  http://www.intmath.com/

- Online applets
  www.explorelearning.org
  www.shodor.org

- Drexel Math Forum
  http://mathforum.org/library/toc.html

- Free Online Calculator
  http://www.coolmath.com/graphit/
  http://www.meta-calculator.com/

- Free online graph paper
  http://incompetech.com/graphpaper/lite/
  http://www.printfreegraphpaper.com/

Content Area/ Grade Level/ Course:	Mathematics / 11 - 12 / PreCalculus
Unit Plan Title:	Unit 3: Analytic Trigonometry
Time Frame	Enriched: 27 days Regular: 32 days

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Algebra

- Functions: Trigonometric Functions, F -TF

Unit Overview

In this unit, students will derive various trigonometric identities. They will be able to use these identities and their knowledge of evalutating trigonometric fuctions to solve trigonometric equations. Students will also be able to verify complex trigonometric identities as well as derive other formulas, such as the sum and difference formulas.

Standard Number(s)   * i.e: Math: F.LE.A.4   i.e.: NJSLSA.R4.

- F -TF.C.8 Prove the Pythagorean identity sin^2 (θ) + cos^2 (θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
- F -TF.C.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems
- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and
- support different interpretations of real-world phenomena.
- 8.1.12.DA.6: Create and refine computational models to better represent the relationships among
- different elements of data collected from a phenomenon or process.
- 8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original

- and existing algorithms.
- 9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12 prof.CR3.a)
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- Standards for Mathematical Practice
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  - K-12.MP.7 Look for and make use of structure.
  - K-12.MP.8 Look for and express regularity in repeated reasoning.

Intended Outcomes - {Essential Questions}

- How are the fundamental trigonometric identities used to evaluate trigonometric functions, simplify trigonometric expressions and rewrite trigonometric expressions?
- How are trigonometric identities verified?

- How do you use standard algebraic techniques and inverse trig functions to solve trigonometric equations?
- How do sum and difference formulas enable one to evaluate trig functions, verify identities and to solve trigonometric equations?
- How are sum and difference formulas, multiple angle formulas, power reducing formulas and product-sum formulas used to simplify, solve, or evaluate trigonometric functions?

Enduring Understandings

- Use fundamental trigonometric identities to evaluate trigonometric functions and simplify trigonometric expressions
- Verify trigonometric identities
- Use standard algebraic techniques and inverse trigonometric functions to solve trigonometric equations
- Use sum and difference formulas, multiple-angle formulas, power-reducing formulas, half-angle formulas, and product-sum formulas to rewrite and evaluate trigonometric functions

Check all that apply. **21st Century Themes**		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills**	
	Global Awareness	E, T, A	**Creativity and Innovation**
	Environmental Literacy	E, T, A	**Critical Thinking and Problem Solving**
X	**Health Literacy**	E, T, A	**Communication**
	Civic Literacy	E, T, A	**Collaboration**
X	**Financial, Economic, Business, and Entrepreneurial Literacy**		

Student Learning Targets/Objectives (Students will know/Students will understand)

Students will be able to:

- simplify trigonometric expressions.

Example:

$$\frac{\tan y + \cot y}{\sec y \csc y}$$

$$\textit{Solution.}\quad \frac{\tan y + \cot y}{\sec y \csc y} = \frac{\frac{\sin y}{\cos y} + \frac{\cos y}{\sin y}}{\frac{1}{\cos y} \cdot \frac{1}{\sin y}} = \frac{\frac{\sin^2 y + \cos^2 y}{\cos y \sin y}}{\frac{1}{\cos y \sin y}} = \sin^2 y + \cos^2 y = 1$$

Answer: 1

$$1 - \frac{\cos^2\theta}{1+\sin\theta}$$

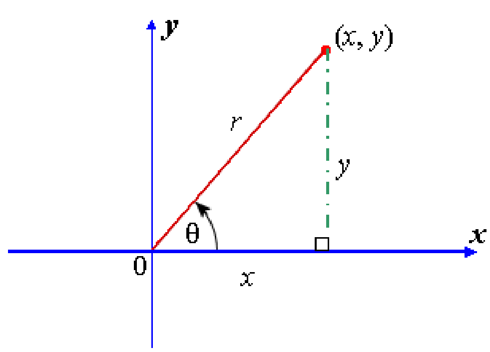
$$\begin{aligned}\textit{Solution.}\quad 1 - \frac{\cos^2\theta}{1+\sin\theta} &= \frac{1+\sin\theta - \cos^2\theta}{1+\sin\theta} \\ &= \frac{\sin^2\theta + \sin\theta}{1+\sin\theta} = \frac{\sin\theta(1+\sin\theta)}{1+\sin\theta} = \sin\theta\end{aligned}$$

Answer: $\sin\theta$

- prove and apply trigonometric identities.

Example: For the diagram indicated below we can see that $sin\ \theta = \frac{y}{r}$ and $cos\ \theta = \frac{x}{r}$. Using the Pythagorean Theorem, we know that $x^2 + y^2 = r^2$. Dividing this equation by r^2, we obtain $\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$.

Connecting $sin\ \theta = \frac{y}{r}$ and $cos\ \theta = \frac{x}{r}$ with $\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$, what can you conclude?



(see Example* above for connection)

- verify trigonometric identities.

Example:

Prove the following trigonometric identity is true:

$$2\csc\theta = \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\csc\theta$$

$$2\csc\theta = \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$$

$$2\csc\theta = \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)(\sin\theta)} \quad \boxed{\text{Combine quotients.}}$$

$$2\csc\theta = \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{(1+\cos\theta)(\sin\theta)} \quad \boxed{\text{Simplify } (1+\cos\theta)^2}$$

$$2\csc\theta = \frac{(\sin^2\theta + \cos^2\theta) + 1 + 2\cos\theta}{(1+\cos\theta)(\sin\theta)} \quad \boxed{\text{Regroup.}}$$

$$2\csc\theta = \frac{2 + 2\cos\theta}{(1+\cos\theta)(\sin\theta)} \quad \boxed{\text{Pythagorean Identity}}$$

$$2\csc\theta = \frac{2(1+\cos\theta)}{(1+\cos\theta)(\sin\theta)} \quad \boxed{\text{Factor}}$$

$$2\csc\theta = \frac{2}{\sin\theta} \quad \boxed{\text{Reduce}}$$

$$2\csc\theta = 2\csc\theta \quad \boxed{\text{Reciprocal Identity}}$$

- solve trigonometric equations.

Example:

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0 \quad \text{(factor)}$$

$$2\sin x - 1 = 0 \quad \text{or} \quad \sin x + 1 = 0$$

$$\sin x = \frac{1}{2} \qquad \sin x = -1$$

$$x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \qquad x = \frac{3\pi}{2}$$

$$x = 30°,\ 150° \qquad x = 270°$$

- use double angle formulas to rewrite and evaluate trigonometric expressions.

Example:

$$\cos 2x - \cos x = 0$$

$$2\cos^2 x - 1 - \cos x = 0$$

$$2\cos^2 x - \cos x - 1 = 0 \quad \Big| \quad 2a^2 - a - 1$$

$$(2\cos x + 1)(\cos x - 1) = 0 \quad \Big| \quad (2a + 1)(a - 1)$$

$2\cos x + 1 = 0$	$\cos x - 1 = 0$
$\cos x = -½$	$\cos x = 1$
$\cos^{-1}(½) = 60^0$	
$x = 120 \text{ or } 240$	$x = 0, 360^0$

$$x = 0^0,\ 120^0,\ 240^0 \text{ or } 360^0$$

Example:

Find all solutions for x on the interval $[0, 360°)$.

$$\tan(2x) = -\frac{1}{2}$$

Assessments (Pre, Formative, Summative, Other)	*Denote required common assessments with an **

Assessments – pretests, formative, summative – are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:

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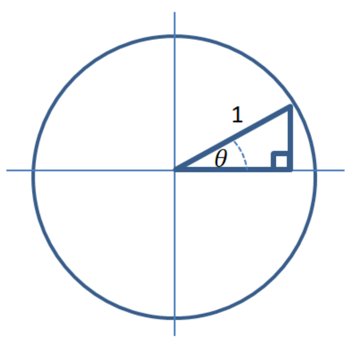
    I. Use of communicators
    II. Group work
    III. Clickers
    IV. Exit Cards
    V. Homework
    VI. Teacher quizzes/tests
    VII. *Common Core Midterm and Final Examinations

Teaching and Learning Activities

Activities	Sample Questions: 1) Use the diagram to explain why Pythagorean Identity "$\sin^2\theta + \cos^2\theta = 1$" is true.  2) Verify the following identities: $\sin x \tan x + \cos x = \sec x$ $\frac{\cos\alpha\sec\alpha}{1+\tan^2\alpha} = \cos^2\alpha$ 3)

Find the values below given the following: $\tan x = \frac{4}{3}$, $180° < x < 360°$

$\cos y = \frac{4}{5}$, $0° < y < 180°$

a) $\sin(x + y) =$ _______ b) $\cos(x + y) =$ _______ c) $\tan(x + y) =$ _______

4)

Find all solutions for x on the interval $[0, 360°)$.

$\tan(2x) = \cot x$ $3\cos^2 x = 4 + 4\sin x$

5)

Which of the following equations has no solutions?

- $\sec x = \frac{1}{3}$
- $\cos x = 2$
- $\sin x = 3$
- $\cot x = \frac{\sqrt{3}}{3}$
- $\tan x = \frac{\sqrt{3}}{3}$
- $\sec x = 2$
- $\sin x = 1$
- $\csc x = 5$

6)

Describe the mistake made in the work shown below.

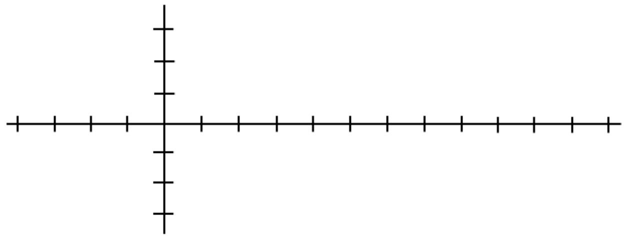
$$\sin 15° = \sin(45° - 30°)$$

$$= \sin 45° - \sin 30°$$

$$= \frac{\sqrt{2}}{2} - \frac{1}{2}$$

$$= \frac{\sqrt{2}-1}{2}$$

7)

	Find all solutions in the interval $[0, 2\pi)$: $2\sin x\cos x + \cos x = 0$ a. $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ b. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ c. $\frac{5\pi}{6}, \frac{11\pi}{6}$ d. $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$ 
Differentiation Strategies	• Alternative Assement projects to demonstrate mastery • Peer to Peer tutoring • implementations of visual representations • Technology implementation, use of graphing calcultor • Allow students to work in small groups • Provide opportunity for questions Differentiation Strategies for Special Education Students Differentiation Strategies for Gifted and Talented Students Differentiation Strategies for ELL Students Differentiation Strategies for At Risk Students
Honors	N/A (See Unified Mathematics 4 Curriculum)

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  www.shodor.org
- Drexel Math Forum
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- Free Online Calculator
  http://www.coolmath.com/graphit/
  http://www.meta-calculator.com/
- Free online graph paper
  http://incompetech.com/graphpaper/lite/
  http://www.printfreegraphpaper.com/

Content Area/ Grade Level/ Course:	Mathematics / 11 - 12 / PreCalculus
Unit Plan Title:	Unit 4: Triangle Trigonometry
Time Frame	Enriched: 12 days Regular: 15 days

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Algebra

- Geometry: Similiarity, Right Triangles, and Trigonometry, G -SRT

Unit Overview

In this unit, students will expand their knowledge of geometry. Using knowledge of right triangle trigonometry, students will derive the Law of Sines. Students will then be able to solve for the sides and angles of oblique triangles using both Law of Sines or Law of Cosines. Alternate area formulas for triangles will also be explored. Finally, students will also be able to draw bearings in order to apply their knowledge of solving triangles to application word problems.

Standard Number(s)   * i.e: Math: F.LE.A.4   i.e.: NJSLSA.R4.

- G -SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- G -SRT.D.9 (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- G -SRT.D.10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
- G-SRT.D.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and
- support different interpretations of real-world phenomena.
- 8.1.12.DA.6: Create and refine computational models to better represent the relationships among
- different elements of data collected from a phenomenon or process.
- 8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original
- and existing algorithms.
- 9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
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- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

- RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
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- RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

- Standards for Mathematical Practice
  - K-12.MP.1 Make sense of problems and persevere in solving them.
  - K-12.MP.2 Reason abstractly and quantitatively.
  - K-12.MP.3 Construct viable arguments and critique the reasoning of others.
  - K-12.MP.4 Model with mathematics
  - K-12.MP.5 Use appropriate tools strategically.
  - K-12.MP.6 Attend to precision
  - K-12.MP.7 Look for and make use of structure.
  - K-12.MP.8 Look for and express regularity in repeated reasoning.

Intended Outcomes - {Essential Questions}

- Under what conditions are the Law of Sines and Law of Cosines used?
- How is the “ambiguous” case solved?
- How are bearings written?
- How can bearings be used to create triangles?
- What are alternative ways to find the area of a triangle?

Enduring Understandings

- Use the Law of Sines and the Law of Cosines to solve oblique triangles
- Determine whether one, two or no triangles can be created in the ambiguous case using Law of Sines
- Use Law of Sines and Cosines to solve triangles created by word problems including bearings
- Find areas of oblique triangles

Check all that apply. **21st Century Themes**		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills**	
	Global Awareness	E, T, A	**Creativity and Innovation**

	Environmental Literacy	E, T, A	**Critical Thinking and Problem Solving**
X	**Health Literacy**	E, T, A	**Communication**
X	**Civic Literacy**	E, T, A	**Collaboration**
X	**Financial, Economic, Business, and Entrepreneurial Literacy**		

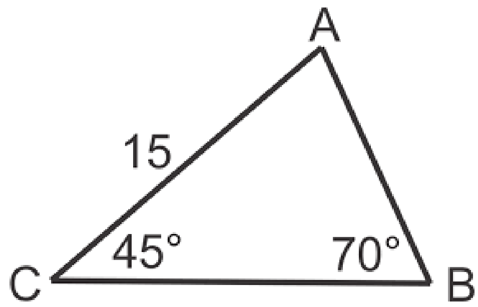
Student Learning Targets/Objectives (Students will know/Students will understand)

Students will be able to :

- use the law of sines and cosines to solve oblique triangles

  Example:

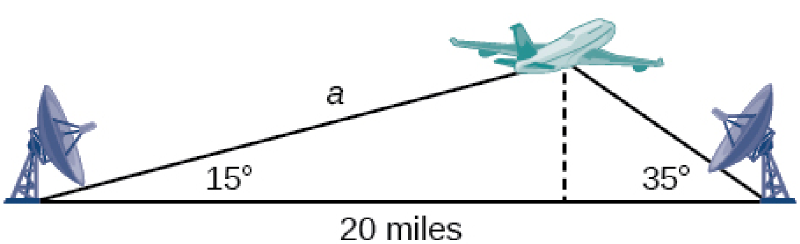
  Find the missing sides and angles in the triangle below:



- use bearings to solve application word problems using Law of Sines and Law of Cosines.

  Example:

  An airplane leaves the airport. Some time later, two radar stations detect the plane. The first radar stationg detects the plane with a bearing of $N\ 75^{o}\ E$ and the second with a bearing of $N\ 55^{o}\ W$. If the radar stations are 20 miles apart and are directly east - west from eachother, how far is the plane from the first radar station?

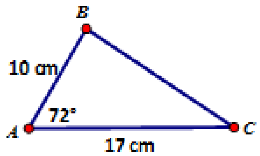


- calculate the areas of oblique triangles.

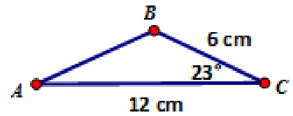
  Example:

Find the area of each of the triangles below:

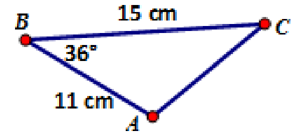
a)



b)



c)



Solutions:

$$Area = \frac{1}{2}bc(Sin\,A)$$
$$Area = \frac{1}{2}(17)(10)(Sin\,72^{\circ})$$
$$Area = 80.84\,cm^2$$

$$Area = \frac{1}{2}ab(Sin\,C)$$
$$Area = \frac{1}{2}(6)(12)(Sin\,23^{\circ})$$
$$Area = 14.07\,cm^2$$

$$Area = \frac{1}{2}ac(Sin\,B)$$
$$Area = \frac{1}{2}(15)(11)(Sin\,36^{\circ})$$
$$Area = 48.49\,cm^2$$

Assessments (Pre, Formative, Summative, Other) ***Denote required common assessments with an ****

Assessments – pretests, formative, summative – are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:

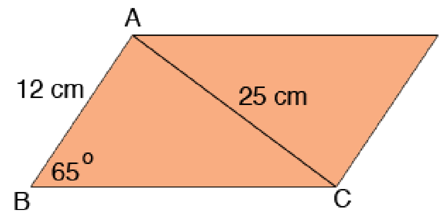
A. performance based tasks in support of activities for essential understanding of objectives.
   I. Projects
   II. Reports
   III. Investigations
   IV. Research

B. other evidence of student learning.
   I. Class Participation
   II. *Benchmark Tests/Quizzes
   III. Teacher Observations

C. varied types of assessment measures to be employed, including rubrics.
   I. Use of communicators
   II. Group work
   III. Clickers
   IV. Exit Cards
   V. Homework
   VI. Teacher quizzes/tests
   VII. *Common Core Midterm and Final Examinations

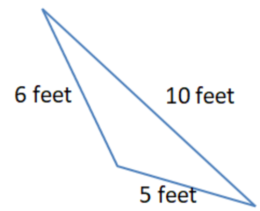
Teaching and Learning Activities

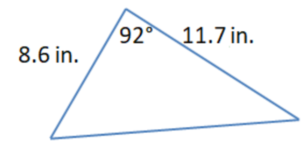
Activities	Sample Questions: 1) Find the length of BC.



2)

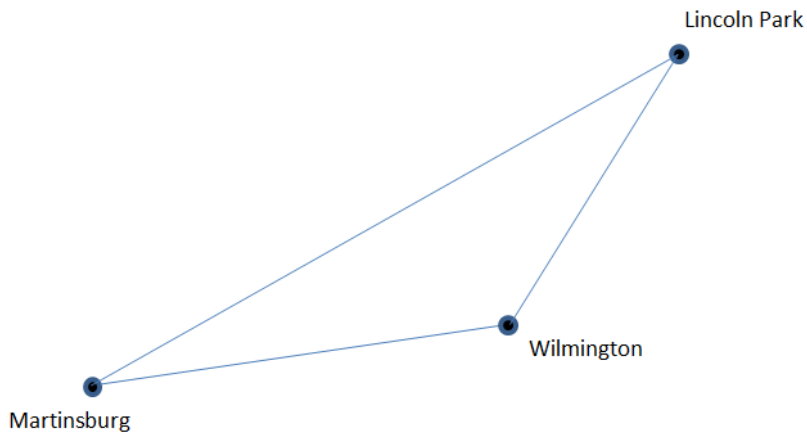
Find the areas of the two triangles below.





3)

A plane leaves Lincoln Park and flies 100 miles with a bearing of S 34° W. It lands in Wilmington, Delaware to refuel. It then flies 130 miles with a bearing of S 81° W to Martinsburg, Maryland. How far is Martinsburg from Lincoln Park?



4)

Given the following information about ΔABC, decide whether there is no solution, exactly one solution, or two solutions. Then justify your answer. You do not need to solve the triangle.

B = 30°, b = 40, a = 60

Differentiation Strategies	• Alternative Assement projects to demonstrate mastery • Peer to Peer tutoring • implementations of visual representations • Technology implementation, use of graphing calcultor • Allow students to work in small groups • Provide opportunity for questions Differentiation Strategies for Special Education Students Differentiation Strategies for Gifted and Talented Students Differentiation Strategies for ELL Students Differentiation Strategies for At Risk Students
Honors	N/A (See Unified Mathematics 4 Curriculum)

Resources

- Online applets and interactive virtual manipulatives
  http://nlvm.usu.edu/en/nav/vLibrary.html

- Interactive Math
  http://www.cut-the-knot.org/index.shtml
  http://www.intmath.com/

- Online applets
  www.explorelearning.org
  www.shodor.org

- Drexel Math Forum
  http://mathforum.org/library/toc.html

- Free Online Calculator
  http://www.coolmath.com/graphit/
  http://www.meta-calculator.com/

- Free online graph paper
  http://incompetech.com/graphpaper/lite/
  http://www.printfreegraphpaper.com/

- Law of Sines Calculator
  https://www.calculatorsoup.com/calculators/geometry-plane/triangle-law-of-sines.php

Content Area/ Grade Level/ Course:	Mathematics / 11 - 12 / PreCalculus
Unit Plan Title:	Unit 5: Polar Coordinates and Graphs of Polar Equations
Time Frame	Enriched: 23 days Regular: ---

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Algebra

- Number and Quantity: Complex Number System - N -CN

Unit Overview

In this unit, students will explore the polar representation of a point. Discovering conversion equations through geometry and trigonometry, students will then be able convert equations to polar form. Expanding this knowledge to the complex number system, students will be able to perform operations with complex polar numbers. In doing this, students will make discoveries about whether polar or rectangular number systems are more conducive for certain operations. Throughout this unit, graphical representations will be used to visualize polar representations.

Standard Number(s)   * i.e: Math: F.LE.A.4   i.e.: NJSLSA.R4.

- N -CN.B.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
- N -CN.B.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, (-1 + √3i)3 = 8 because (-1+ √3i) has modulus 2 and argument 120°.
- N -CN.B.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and
- support different interpretations of real-world phenomena.
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    - K-12.MP.8 Look for and express regularity in repeated reasoning.

Intended Outcomes - {Essential Questions}

- How do you plot points in the polar coordinate system?
- How can we find multiple representations of points in the polar coordinate system?
- How do we convert points and equations from rectangular to polar form?
- How do we convert points and equations from polar form to rectangular form?
- How do we graph a polar equation by hand?
- How do we use symmetry, zeros, and maximum “r” values as graphing aids?
- What are the names of the special polar graphs and how can we identify them?
- How is the “absolute value” of a complex number found and what are the “modulus and argument”?
- How and why are complex numbers in trigonometric form multiplied and divided?
- How is DeMoivre’s Theorem used to find powers and nth roots of complex numbers?
- How can a graphing utility be used to find the nth roots of a complex number?

Enduring Understandings

- Plot points and find multiple representations of points in the polar coordinate system
- Convert points from rectangular to polar form and vice versa
- Convert equations from rectangular to polar form and vice versa
- Graph polar equations and recognize special polar graphs

- Multiply and divide complex numbers written in trigonometric form
- Find powers and nth roots of complex numbers

Check all that apply. **21st Century Themes**		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills**	
	Global Awareness	E, T, A	**Creativity and Innovation**
	Environmental Literacy	E, T, A	**Critical Thinking and Problem Solving**
X	**Health Literacy**	E, T, A	**Communication**
	Civic Literacy	E, T, A	**Collaboration**
X	**Financial, Economic, Business, and Entrepreneurial Literacy**		

Student Learning Targets/Objectives (Students will know/Students will understand)

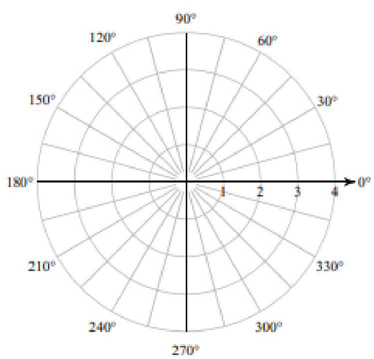
Students will be able to:

- plot points and find multiple representations of points in the polar coordinate system.

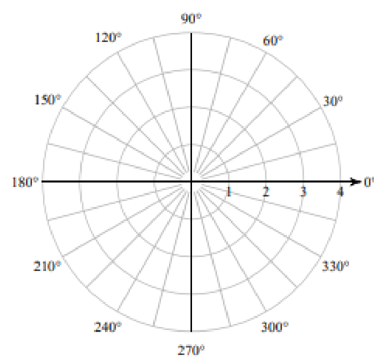
Example:

Plot the point with the given polar coordinates.

$(3, 150°)$



$\left(-3, \frac{23\pi}{12}\right)$



- convert points and equations from rectangular to polar form and vice versa.

Example:

1. Convert the polar equation to rectangular coordinates. (Use variables x and y as needed.)

$$\theta = \frac{\pi}{2}$$

2. Convert the equation to polar form. (Use variables r and θ as needed.)

$$y = 5x^2$$

3. Find the rectangular coordinates for the point whose polar coordinates are given.

$$\left(8\sqrt{2}, -\frac{\pi}{4}\right)$$

- Graph polar equations and recognize special polar graphs

Example:

Identify the horizontal and vertical intercepts of the limaçon represented by the equation $r = 7 + 3\cos\theta$.

vertical intercepts: $\left(7, \frac{\pi}{2}\right)$ and $\left(-7, \frac{\pi}{2}\right)$
horizontal intercepts: $(10, 0)$ and $(-4, 0)$

vertical intercepts: $\left(7, \frac{\pi}{2}\right)$ and $\left(-7, \frac{\pi}{2}\right)$
horizontal intercepts: $(10, 0)$ and $(4, 0)$

vertical intercepts: $\left(-10, \frac{\pi}{2}\right)$ and $\left(4, \frac{\pi}{2}\right)$
horizontal intercepts: $(7, 0)$ and $(-7, 0)$

vertical intercepts: $\left(10, \frac{\pi}{2}\right)$ and $\left(-4, \frac{\pi}{2}\right)$
horizontal intercepts: $(7, 0)$ and $(-7, 0)$

Example:

Classify the limaçon represented by the equation $r = \frac{1}{2} - \frac{1}{2}\sin\theta$.

cardioid
dimpled
convex
inner loop

- multiply and divide complex numbers written in trigonometric form.

Example:

Let $z = 3\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$, $w = 6\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$. Perform each indicated operation. Leave your answers in polar form. Leave your arguments in radians.

(a) Find zw.

(b) Find $\frac{z}{w}$.

- find powers and nth roots of complex numbers.

Example:

Find $(1 + i\sqrt{3})^3$ using De Moivre's Theorem.

SOLUTION

The argument of $z = 1 + i\sqrt{3}$ is $\theta = \pi/3$, and its modulus is $|1 + i\sqrt{3}| = \sqrt{1 + 3} = 2$. Therefore,

$$z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$\begin{aligned} z^3 &= 2^3\left[\cos\left(3\cdot\frac{\pi}{3}\right) + i\sin\left(3\cdot\frac{\pi}{3}\right)\right] \\ &= 8(\cos\pi + i\sin\pi) \\ &= 8(-1 + 0i) = -8 \end{aligned}$$

Assessments (Pre, Formative, Summative, Other) ***Denote required common assessments with an ****

Assessments – pretests, formative, summative – are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:

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III. Teacher Observations

C. varied types of assessment measures to be employed, including rubrics.

I. Use of communicators
II. Group work
III. Clickers
IV. Exit Cards
V. Homework
VI. Teacher quizzes/tests
VII. *Common Core Midterm and Final Examinations

Teaching and Learning Activities

Activities

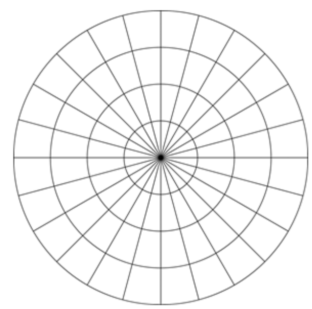
Sample Questions:

1)

Graph the polar point $\left(6, \frac{3\pi^{r}}{4}\right)$ on the grid to the right.

Then, give another name for the point in question in polar coordinates and then convert the point to rectangular coordinates.

Polar: ______________ Rectangular: ______________



2)

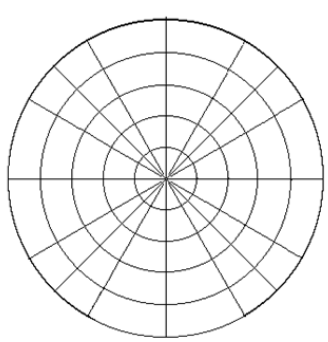
Convert the following to polar equations.

$x^2 + y^2 - 3y = 0$ $x = -4$

3)

Graph the following polar equations after sketching the graph on a Cartesian plane. Then algebraically find all of the points of intersection. Give all points with exact answers.

$r = 2 + 2\sin\theta$ $r = 2 - 2\cos\theta$



4)

	Convert the following numbers to rectangular if it is given in polar, or vice versa. $z_1 = 4\ \text{cis}\,\frac{7\pi}{4}$ $\qquad$ $z_2 = 7 - 2i$ $z_3 = -1 + \sqrt{3}\,i$ $\qquad$ $z_4 = 7\ \text{cis}\,\frac{4\pi}{3}$ Perform the operations and answer the questions using $z_1 - z_4$ from above. $z_1 * z_4$ (using polar) $\qquad$ $z_2 * z_3$ (using rectangular)
Differentiation Strategies	• Alternative Assement projects to demonstrate mastery • Peer to Peer tutoring • implementations of visual representations • Technology implementation, use of graphing calcultor • Allow students to work in small groups • Provide opportunity for questions Differentiation Strategies for Special Education Students Differentiation Strategies for Gifted and Talented Students Differentiation Strategies for ELL Students Differentiation Strategies for At Risk Students
Honors	N/A (See Unified Mathematics 4 Curriculum)

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  http://www.intmath.com/
- Online applets
  www.explorelearning.org
  www.shodor.org
- Drexel Math Forum
  http://mathforum.org/library/toc.html

- Free Online Calculator
  http://www.coolmath.com/graphit/
  http://www.meta-calculator.com/

- Free online graph paper
  http://incompetech.com/graphpaper/lite/
  http://www.printfreegraphpaper.com/

Content Area/ Grade Level/ Course:	Mathematics / 11 - 12 / PreCalculus
Unit Plan Title:	Unit 6: Limits and an Introduction to Calculus
Time Frame	Enriched: 5 days Regular: ---

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Algebra

- Functions: Interpreting Functions, F -IF

Unit Overview

This unit will only be used if all other material has been sufficiently covered. Students will revisit well-known functions in order to analyze limits at given values and to infinity. Various methods, such as direct substition and estimation, will be addressed. Students will also discuss limits of sequences.

Standard Number(s)   * i.e: Math: F.LE.A.4   i.e.: NJSLSA.R4.

- F -IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
- F -IF.C.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- F -IF.C.7d (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
- F -IF.C.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and
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- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

- RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
- RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
- RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

- Standards for Mathematical Practice
  - K-12.MP.1 Make sense of problems and persevere in solving them.
  - K-12.MP.2 Reason abstractly and quantitatively.
  - K-12.MP.3 Construct viable arguments and critique the reasoning of others.
  - K-12.MP.4 Model with mathematics
  - K-12.MP.5 Use appropriate tools strategically.
  - K-12.MP.6 Attend to precision
  - K-12.MP.7 Look for and make use of structure.
  - K-12.MP.8 Look for and express regularity in repeated reasoning.

Intended Outcomes - {Essential Questions}

- What is the definition of a limit?
- How do we estimate limits and how do we determine if a limit exists?
- How do we use algebraic, graphing and numerical techniques to find the limit of a function?
- How do we evaluate one-sided limits?
- How do we evaluate limits of difference quotients from calculus?
- What are real life applications of limits?

Enduring Understandings

- Estimate limits and use properties and operations of limits
- Find limits by direct substitution and by using the dividing out and rationalizing techniques
- Evaluate limits at infinity and find limits of sequences

Check all that apply. **21st Century Themes**		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills**	
	Global Awareness	E, T, A	**Creativity and Innovation**

	Environmental Literacy	E, T, A	Critical Thinking and Problem Solving
X	Health Literacy	E, T, A	Communication
X	Civic Literacy	E, T, A	Collaboration
X	Financial, Economic, Business, and Entrepreneurial Literacy		

Student Learning Targets/Objectives (Students will know/Students will understand)

Students will be able to:

- estimate limits and use properties and operations of limits.

Example:

$$\begin{aligned}\lim_{x\to-2}\left(3x^2+5x-9\right) &= \lim_{x\to-2} 3x^2 + \lim_{x\to-2} 5x - \lim_{x\to-2} 9 \\ &= 3\lim_{x\to-2} x^2 + 5\lim_{x\to-2} x - \lim_{x\to-2} 9\end{aligned}$$

- find limits by direct substitution and by using the dividing out and rationalizing techniques.

Example:

Find the limit: $\lim_{x\to-3} \dfrac{x^2+x-6}{x+3}$.

Solution:

From the discussion above, you know that direct substitution fails.

So, begin by factoring the numerator and dividing out any common factors.

$$\lim_{x\to-3} \frac{x^2+x-6}{x+3} = \lim_{x\to-3} \frac{(x-2)(x+3)}{x+3}$$ Factor numerator.

- evaluate limits at infinity and find limits of sequences.

Example:

Find the limit of the sequence whose nth term is

$$a_n = \left(1 + \frac{1}{n}\right)^n.$$

Solution:

You learned that $\lim\limits_{x\to\infty} \left(1 + \frac{1}{x}\right)^x = e.$

So, you can apply Theorem 1 to conclude that

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$$
$$= e.$$

Assessments (Pre, Formative, Summative, Other) ***Denote required common assessments with an ****

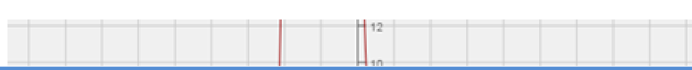
Assessments – pretests, formative, summative – are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:

A. performance based tasks in support of activities for essential understanding of objectives.
   I. Projects
   II. Reports
   III. Investigations
   IV. Research

B. other evidence of student learning.
   I. Class Participation
   II. *Benchmark Tests/Quizzes
   III. Teacher Observations

C. varied types of assessment measures to be employed, including rubrics.
   I. Use of communicators
   II. Group work
   III. Clickers
   IV. Exit Cards
   V. Homework
   VI. Teacher quizzes/tests
   VII. *Common Core Midterm and Final Examinations

Teaching and Learning Activities

Activities	Sample Questions: 1) Complete the limit statements below, given the graph of f(x): $f(x) = \frac{(x-3)(x+5)}{x(x-3)(x+2)}$ $\lim f(x) =$ 

	2) **Find the limit algebraically.** $\lim\limits_{x \to \infty} \frac{x^2}{3x} =$ 3) **Find the limit algebraically.** $\lim_{x\to\infty} 5 + \frac{3x}{x^2} =$
Differentiation Strategies	• Alternative Assement projects to demonstrate mastery • Peer to Peer tutoring • implementations of visual representations • Technology implementation, use of graphing calcultor • Allow students to work in small groups • Provide opportunity for questions Differentiation Strategies for Special Education Students Differentiation Strategies for Gifted and Talented Students Differentiation Strategies for ELL Students Differentiation Strategies for At Risk Students
Honors	N/A (See Unified Mathematics 4 Curriculum)

Resources

- Online applets and interactive virtual manipulatives
  http://nlvm.usu.edu/en/nav/vLibrary.html

- Interactive Math
  http://www.cut-the-knot.org/index.shtml
  http://www.intmath.com/

- Online applets
  www.explorelearning.org
  www.shodor.org

- Drexel Math Forum
  http://mathforum.org/library/toc.html

- Free Online Calculator
  http://www.coolmath.com/graphit/
  http://www.meta-calculator.com/

- Free online graph paper
  http://incompetech.com/graphpaper/lite/
  http://www.printfreegraphpaper.com/